



Lightweight Tandem Bicycle Frame Design

Finite Element Analysis

Harry Schlote

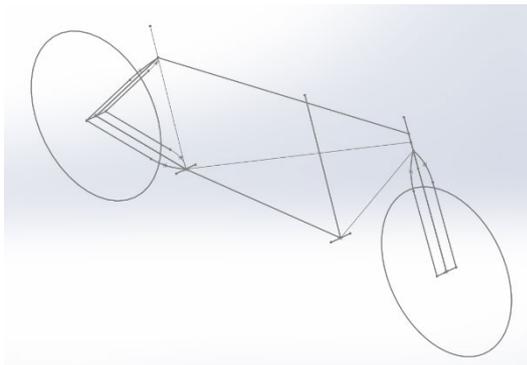


Table of Contents

1. Introduction	2
2. Methods	2
2.1 Methodology.....	2
2.2 Assumptions.....	2
2.2 Boundary Conditions.....	3
2.3 Applying Loads	3
2.4 Fatigue Simulation	3
2.5 Frequency Simulation	4
3. Results	4
3.1 Mesh Refinement.....	4
3.2 Original Design Results (Aluminium).....	5
3.2.1 Frequency Analysis	5
3.2.2 Fatigue Analysis	5
3.3 Iterated Design Results (Aluminium)	6
3.3.1 Frequency Analysis	6
3.3.2 Fatigue Analysis	6
3.4 Original Design Results (Titanium).....	6
3.4.1 Frequency Analysis	7
3.4.2 Fatigue Analysis	7
3.5 Iterated Design Results (Titanium).....	7
3.5.1 Frequency Analysis	7
3.5.2 Fatigue Analysis	7
3.6 Sanity Checks.....	7
4. Discussion	8
4.1 Summary of Results	8
4.2 Limitations.....	8
5. References	9

1. Introduction

The project brief is to design a lightweight frame for a 2-person tandem bicycle. This frame should meet the criteria of having a natural resonant frequency of over 30Hz to avoid discomfort to the user due to whole body vibrations, and also have a lifespan of at least 10 years (which is equivalent to 1 million loading cycles). An iteration of the original frame design is to be made in order to improve the natural frequency. The frames should be tested for fatigue life. The two aluminium frames should then also be tested with titanium as the material and the analysis should be repeated. The frame size has a number of constraints, including a length of between 1.5 to 2 metres, seat joint height of 800mm and should accommodate wheels of 26-inch diameter. Welds of 5mm fillets will be used to join the tubes.



The initial sketch of the frame, using production tandem dimensions^[1] wheels were included to help visualise and dimension the tubing around them

2. Methods

2.1 Methodology

The first design will start with the most lightweight frame. Current frames are around 4 kg^[2] so for this study to be realistic, the mass should not be much larger than this. As well as this, the average wall thicknesses for aluminium bike tubing's are around 0.8 - 1mm^[3] (any

larger is unrealistic) and the average diameter of the tubes are around 38mm^[4]. The second iteration will then aim to increase the resonant frequency compared to the initial design.

2.2 Assumptions

As finite element analysis simulations can never exactly replicate real life environments, a number of assumptions had to be made. It was assumed that the material had no imperfections which would in reality affect the structural integrity of the frame. As well as this, it was assumed that all the joints were perfect and that there was no damage caused to the frame material by external (weather etc) effects. In reality there would be stress concentrations at the grain boundary between the weld and frame as seen in the diagram below.

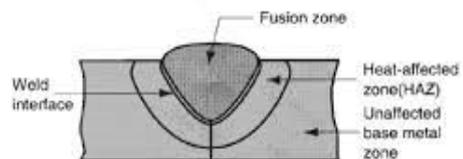
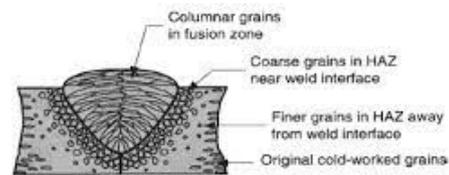
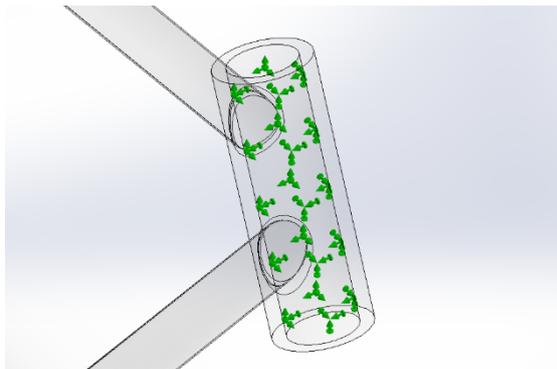


Diagram showing the imperfections in weld joints^[5]

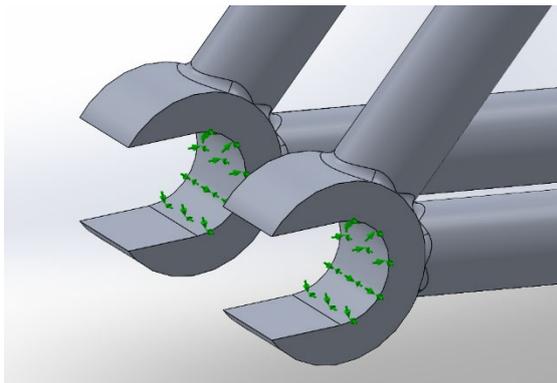
It was also assumed that the stress from the users were the main cause of fatigue and frequency, so drag forces and heat stress and other external forces were ignored. The weight of the cyclists is also unlikely to be evenly distributed, and the force on the pedals in reality would not be constant as we have assumed.

2.3 Boundary conditions

There were two parts of the bike that were fixed. The inner surface of the fork shell was rigidly fixed meaning that it had zero degrees of freedom. The rear wheel bearings are fixed as hinges so that they had only one degree of freedom and could rotate about their axis.



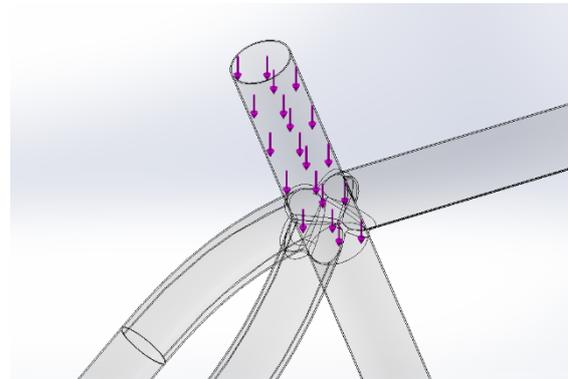
Above image showing the fixed shell, and below showing the rear wheel bearing which can rotate about its axis



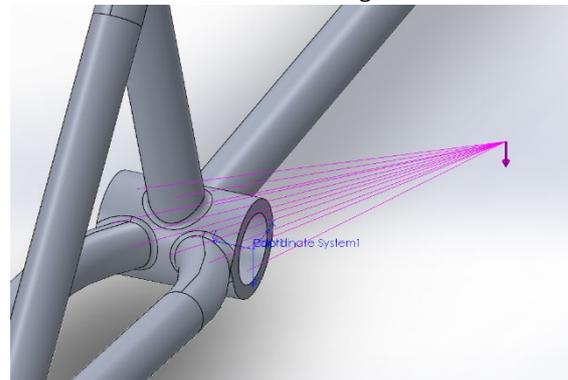
2.4 Applying Loads

There were two types of loads. The force from the two riders of 150kg mass each was multiplied by the value of gravity (9.81) to give a force of 2943N. This acted as a vertical dead load on the two tubes that the 150mm long seat posts would be attached to. This force was applied to the two posts combined. The second forces that oscillated between 0 and 750N were the pedal forces which acted 200mm forward and 100mm sideways from the front and rear crank shells. The remote

load forces had to use coordinate systems on each of the shells and acted vertically downwards.



The vertical force is applied 15mm into the tube, which is the length that a seat post is inserted into a bike on average^[6]



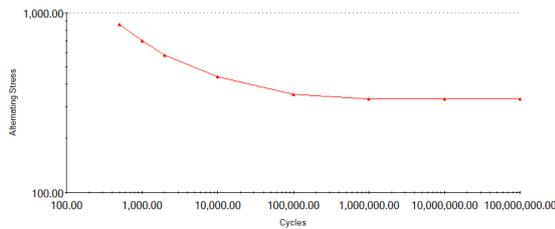
Remote load force was applied to the crankshaft, and the front and rear pedals are in sync with each other due to the chain connecting them on the bike

2.5 Fatigue Simulation

The aim of the fatigue study was to see which parts of the bike were damaged the most after the 1 million cycles which is equivalent to a lifetime of 10 years, and also to find the lifetime of the frame design in number of cycles which depends on the material properties as well as the stress fluctuations. The frame is constructed from Aluminium 7075-T6 which was assumed to be ductile. Titanium Grade 9 which is commonly used for bike frames^[7] was used and as this data wasn't in Solidworks it had to be inputted as a new material. By using values from multiple sources^{[7][8][9][10]},

it improved reliability. Titanium alloy is also considered to be ductile. Therefore, Gerber mean stress correction method can be used for the mean stress correction as the S-N curves in Solidworks are only for R = -1:

$$S_{ca} = \frac{S}{1 - (\sigma_{mean}/S_u)^2}$$



This shows the S-N curve for titanium that had to be manually inputted, along with titanium property values^[7]

As there is the constant force of the user applied, while also the oscillating load of the pedals, then the stress ratio is larger than 0.

$$R = \frac{\sigma_{min}}{\sigma_{max}}$$

Due to the principle of superposition, the constant stress caused by the passengers can be added which causes an increase in the base stress fluctuations. 'Find cycle peaks' method is used as this takes the maximum oscillating stress plus the constant dead load and then the minimum oscillating stress plus the constant dead load and combines them in a single event to find the worst case for fatigue.

2.6 Frequency Simulation

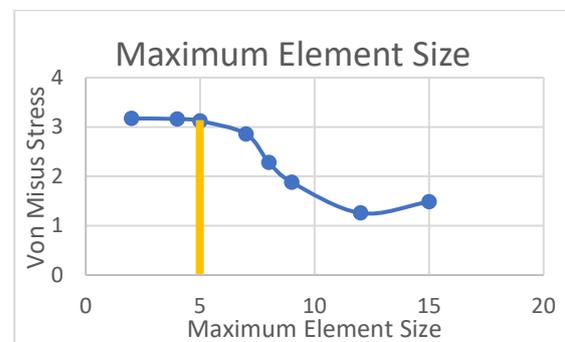
For the frequency simulation, the constant force of 2943 N was applied to the top two posts as well as the two 750N forces on one side of both pedals crank shells. The pedal force on the other side does not need to be analysed as the bike frame is symmetrical. This simulation was also done with only the dead load to simulate

the user only sitting on the bike, but it was found that both tests gave frequencies with negligible differences ($< x10^{-1}$). Tensile loads are known to increase the resonant frequency while compressive loads will reduce it. 'Direct sparse solver' was used as this solves equations directly instead of approximating meaning there is no error in the calculations.

3. Results

3.1 Mesh Refinement

Getting the right type and size of mesh is crucial in order to be able to run studies quickly and accurately. Initially, standard mesh was used, with mesh control applied to the area with stress concentrations. This approach was slow and failed for mesh sizes larger than 7mm as this mesh cannot adapt to the different geometries of the frame such as the sharp edges due to the high aspect ratios. I therefore decided to use blended curved mesh as it adapts to the different regions of the model using an algorithm and adds more elements in areas that require more detail. This is also why this mesh is quicker to simulate. I began with a maximum element size of 15mm and minimum size of 3mm. This mesh size was then refined as seen in the graph to show how the maximum and minimum element size affects the Von Mises stress (which is used in fatigue analysis for ductile materials).

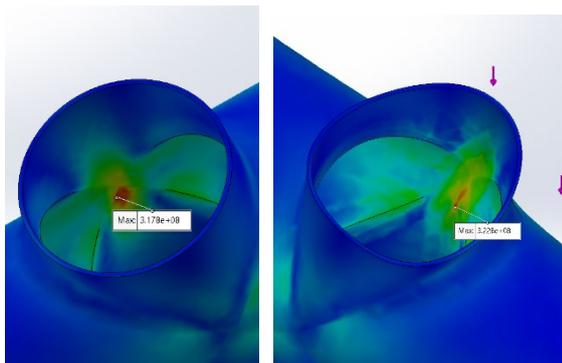


Graph showing that after the maximum element size is less than 5mm, there is

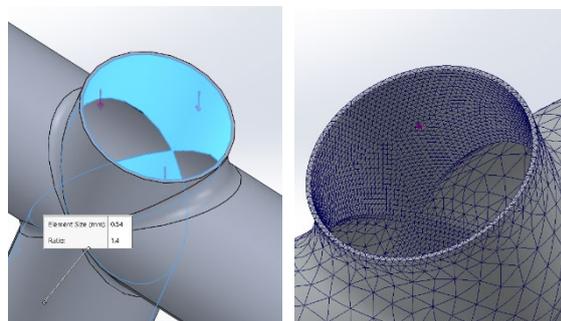
negligible stress change, so this was used (with minimum element size of 1mm)

Once the mesh size was refined and was accurate, a static study was run to locate the areas of high stress concentration. The highest area was around where the force was applied on the seat joints. This made logical sense as the walls are only 0.8mm thick which is less than the welded joints and the shells, meaning the area where the force was directly applied to on these tubes came under the largest stress.

For the second iteration, the mesh setup was identical, as it was a similar structure and for the mesh control it was found that the same locations came under the highest stress.



Above showing the before and after effect from mesh control, which helps pinpoint and give a more accurate reading of stress. Below shows where the mesh control was applied to.



3.2 Original Design Results

The original design as mentioned, had a wall thickness of 0.8mm and external diameter of 38mm, with 36mm diameter tubes used for the 'chain stay' and 'seat

stay' tubes. This made it extremely lightweight at 3.82kg, in line with current aluminium frames on the market.

3.2.1 Frequency Study 1

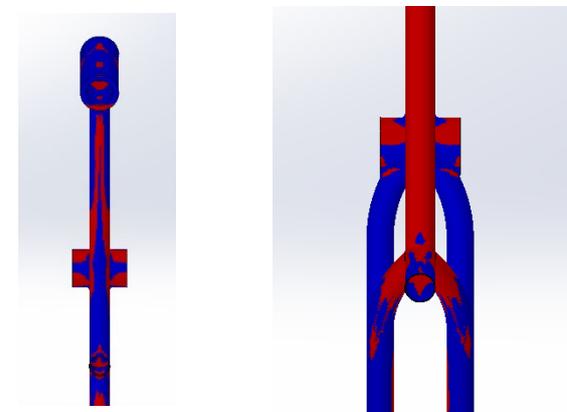
The bike performed well in this study. Its mode shape 1 value was 49.51Hz, which is far above the required 30Hz. From the diagram, the areas in red are most affected. It can be seen that this is around the crankshaft where there are long members which cause resonance at the antinodes. This could be reduced by adding extra structural members which would reduce torsional stress and prevent motion.



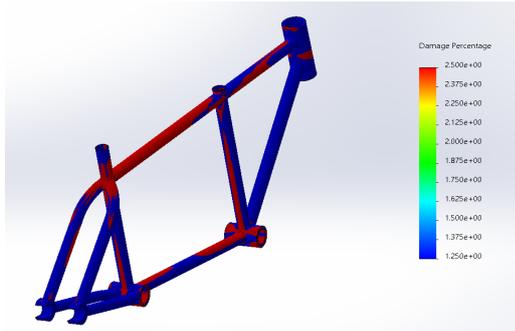
Frequency study visual of the first aluminium design

3.2.2 Fatigue Study 1

For this original design, when the fatigue study was run, the alternating stress was below the S-N curve fatigue limit. This gave a lifetime of 40 million cycles. A diagram showing the regions of higher damage can be seen.



Showing the symmetry of the damage on the frame, and where the higher damage areas are located (red colour)



This shows the overall areas of higher damage

This fatigue analysis showed that the bike fulfilled the 1 million cycles, and it has the maximum lifespan of 40 million cycles. As the Von Misus stress was used (due to ductile material) this was less than the yield stress, so the material didn't fail initially in the static study.

$$\sigma_v > S_y$$

The areas that are highlighted in red could develop microscopic cracks over a longer period of time, so increasing the wall thickness could be a solution to minimise this issue even if the design already fulfils the lifespan.

3.3 Iterated Design Results

The second design had a thicker wall of 1mm, while keeping the external diameter the same at 38mm. A diagonal member connecting the rear pedal crankshaft and the front steering shell was also added. This resulted in a new weight of 5.40kg, which is an increase of 41% compared to the original, but still reasonable compared to other frames on the market.

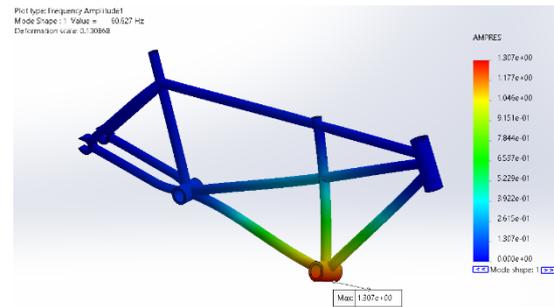
3.3.1 Frequency Study 2

Using the same mesh and loading and boundary conditions, the second frame design was simulated. It was found that the resonant frequency increased to 60.63Hz, an increase of 22%. This is likely due to the structural member, but also the thicker walls of the bike. This gives a larger cross section area and therefore as

stiffness increases with area^[11], the natural frequency will increase.

$$k_a = E \frac{\pi}{4} (d_o^2 - d_i^2)$$

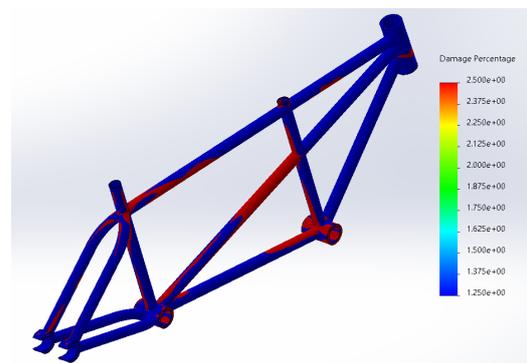
$$\sqrt{k} \propto \omega$$



Iterated design with higher resonant frequency.

3.3.2 Fatigue Study 2

The second fatigue study gave the same maximum lifespan of 40 million cycles as the stress is always below the S-N curve. From the damage visual it can be seen that more of the damage is likely to occur on the main diagonal bar, but from the previous design, it shows the bike would still function if this diagonal bar were to be weakened over time.



Damage visual of iterated design

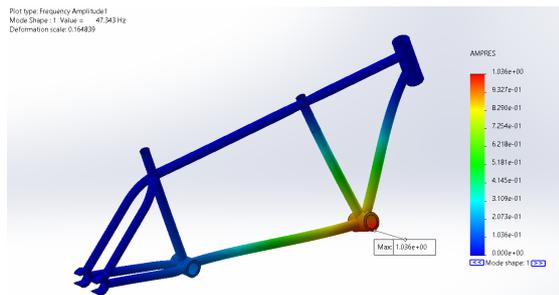
3.4.1 Frequency Study 1 (Titanium)

With more dense titanium now being used it was expected that the frequency would

decrease due to the equation linking mass and frequency:

$$\omega = \sqrt{\frac{k}{m}}$$

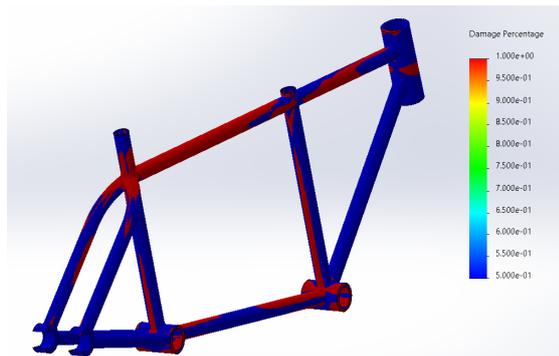
As the structure is the same and only the material is changed, then the frequency will decrease as mass increases. The study gave a value of 47.3Hz compared to the 49.51Hz with aluminium.



Frequency visual of original design with titanium as the material

3.4.2 Fatigue Study 1 (Titanium)

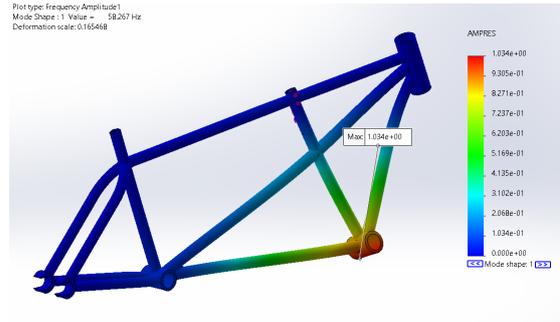
The fatigue study for titanium gives a lifespan of 100 million cycles and maximum damage of 1%. The areas of damage are almost identical to the aluminium study, due to the same structure.



Damage visual of titanium original design

3.5.1 Frequency Study 2 (Titanium)

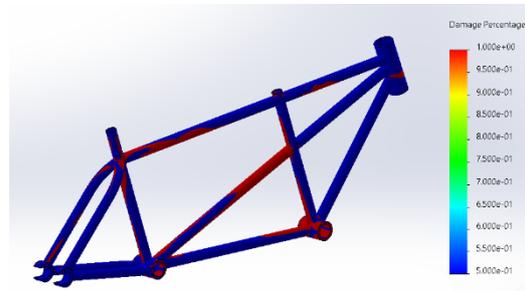
The frequency for the second design with denser titanium again decreased to 58.3Hz compared to the 60.63Hz with aluminium.



Frequency visual of iterated titanium design

3.5.2 Fatigue Study 2 (Titanium)

Again, a similar damage visual of areas with higher damage, and a lifespan of 100 million cycles was found.



Damage visual of the iterated titanium design.

3.6 Sanity Checks

Validation of the fatigue studies were done using the static studies to check that they had a reasonable order of magnitude for Von Mises stress.

The frequency and fatigue life results were checked against other online tandem frame papers to compare if certain methods and magnitudes were similar. This also helped validate the custom S-N curve for titanium.

Multiples of each study were run, and new mesh was created to ensure that results were identical and demonstrated a successful, consistent mesh.

Rather than just reporting results, the visual simulations were checked to see if there was the expected deformation according to FEA theory. The values used

for Grade 9 Titanium were cross checked with multiple other online sources to ensure reliability.

4. Discussion

4.1 Summary of Results

Design	Mass (kg)	Material	Resonant Frequency (Hz)	Fatigue life (cycles)
Original	3.82	Aluminium	49.51	4×10^7
Iterated	5.40	Aluminium	60.63	4×10^7
Original	6.10	Titanium	47.31	1×10^8
Iterated	8.61	Titanium	58.27	1×10^8

With the 4 different bike frames analysed, it can be seen that all of them met the frequency and fatigue life requirements. It was observed that the titanium was heavier, and gave overall lower frequencies than aluminium, but also produced a better fatigue life value. The iterated frame increased the frequency significantly for both materials, but also the mass of the bike. It has an increased frequency 102% higher than the 30Hz required and the fatigue life from 1 million cycles requirement has been increased by 3900%. This bike has met the goal of increasing the resonant frequency but in reality, would not be used in industry as of its heavier weight. The original aluminium frame is the design that would most likely be used as it also far exceeds the design requirements but is also the most lightweight.

4.2 Limitations

A number of factors mean that the results found from the numerous studies are not accurate and limited. The first is that as the frame is an ideal scenario mathematical model, then a real-life frame will show a difference in its response. Due to the large number of nodes in the model, the mesh can only be so fine before it takes too long to compute results which limits the accuracy of results. A safety factor should be used to account for the

lack of random imperfections in the model and this would be used by a manufacturer in industry. The jacobian points value could also be increased from 16 to 29 as this limits the evaluation of a volume integral at special points (such as very curved or sharp edges) inside the element. This is of course if the computational power allows. Other parts of the bike could also be modelled such as the wheels and the front forks, as well as including gravity to account for the force of the bikes weight. These could allow a more accurate application of boundary conditions.



In reality, random occurrences that happen in real life such as dents and corrosion cannot be modelled



The two bike designs, original at the top

5. References

1. Cyclinguk.org. 2021. *Entry-level tandems* | *Cycling UK*. [online] Available at: <<https://www.cyclinguk.org/cycle/entry-level-tandems>> [Accessed 18 April 2021].
2. Tandems.co.uk. 2021. *ORBIT TANDEMS Frame Velocity - Tandem Frames & Forks - Frames - JD Tandems*. [online] Available at: <<https://www.tandems.co.uk/m4b1s8p234/ORBIT-TANDEMS-Frame-Velocity>> [Accessed 19 April 2021].
3. 4. The Tall Cyclist. 2021. *I Like Big Butts: Custom Framebuilding, tube size and butting*. [online] Available at: <<http://www.thetallcyclist.com/2016/09/the-oretical-framebuilding-part-2-tube-size-and-butting/#:~:text=Steel%20to%20aluminum%3A%20An%20aluminum,215%2D260%25%20more%20material.>> [Accessed 19 April 2021].
5. iitg.ac.in. 2021. [online] Available at: <http://www.iitg.ac.in/engfac/ganu/public_html/metal_welding_processes.pdf> [Accessed 20 April 2021].
6. Decathlon.co.uk. 2021. *Seat Post 29.8 mm 350 mm - Black*. [online] Available at: <https://www.decathlon.co.uk/p/seat-post-29-8-mm-350-mm-black/_/R-p-199271?mc=8522030&utm_source=google&utm_medium=organic&utm_campaign=SurfacesForGoogle&utm_content=8522030-2631814&iv=__iv_p_1_g_63052683495_c_316341522215_w_pla-306378754696_n_g_d_c_v_l_t_r_x_pla_y_15177021_f_online_o_8522030-2631814_z_GB_i_en_j_306378754696_s__e_h_9072504_ii__vi__&gclid=Cj0KCQ
7. Cdn.ymaws.com. 2021. [online] Available at: <https://cdn.ymaws.com/titanium.org/resource/resmgr/ZZ-WCTP1984-VOL2/1984_Vol.2-4-The_Development.pdf> [Accessed 21 April 2021].
8. Reilly Cycleworks. 2021. *The grades of Titanium we use*. [online] Available at: <<https://www.reillicycleworks.com/blogs/news/138549703-the-grades-of-titanium-we-use>> [Accessed 21 April 2021].
9. AZoM.com. 2021. *Grade 9 Ti 3Al 2.5V Alloy (UNS R56320)*. [online] Available at: <<https://www.azom.com/article.aspx?ArticleID=9337>> [Accessed 21 April 2021].
10. Elgiloy Specialty Metals |. 2021. *Grade 9 Titanium Tubing Ti-3-2.5 Alloy - Titanium Strips* | *Elgiloy Specialty Metals*. [online] Available at: <<https://www.elgiloy.com/strip-titanium-grade-9-ti-3-25/>> [Accessed 21 April 2021].
11. Orcina.com. 2021. *Homogeneous pipe*. [online] Available at: <<https://www.orcina.com/webhelp/OrcaFlex/Content/html/Homogeneouspipe.htm>> [Accessed 21 April 2021].